

Unruh radiation and Interference effect *

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Abstract

A uniformly accelerated charged particle feels the vacuum as thermally excited and fluctuates around the classical trajectory. Then we may expect additional radiation besides the Larmor radiation. It is called Unruh radiation. In this report, we review the calculation of the Unruh radiation with an emphasis on the interference effect between the vacuum fluctuation and the radiation from the fluctuating motion. Our calculation is based on a stochastic treatment of the particle under a uniform acceleration. The basics of the stochastic equation are reviewed in another report in the same proceeding [2]. In this report, we mainly discuss the radiation and the interference effect.

STOCHASTIC ALD EQUATION

The Unruh radiation is the additional radiation expected to be emanated by a uniformly accelerated charged particle [3]. A uniformly accelerated observer feels the quantum vacuum as thermally excited with the Unruh temperature $T_U = \hbar a / 2\pi c k_B$. Hence as the ordinary Unruh-de Wit detector, a charged particle interacting with the radiation field can be expected to fluctuate around the classical trajectory. Is there additional radiation associated with this fluctuating motion? It is the issue of the present report.

In order to formulate the dynamics of such fluctuating motion, we make use of the stochastic technique. Namely, we solve a set of equations of the accelerated particle and the radiation field in a semiclassical approximation. By semiclassical, we mean that the radiation field is treated as a quantum field while the particle is treated classically.

Since the accelerated particle dissipates its energy through the Larmor radiation, the equation of motion contains the radiation damping term. This is the Abraham-Lorentz-Dirac (ALD) equation. Furthermore, since the accelerated particle feels the Minkowski vacuum as thermally excited, a noise term is also induced in the equation of motion. The stochastic equation of the accelerated charged particle is called the stochastic ALD equation and derived by [4].

We consider the scalar QED whose action is given by

$$S[z, \phi, h] = -m \int d\tau \sqrt{\dot{z}^\mu \dot{z}_\mu} + \int d^4x \frac{1}{2} (\partial_\mu \phi)^2 + \int d^4x j(x; z) \phi(x). \quad (1)$$

where

$$j(x; z) = e \int d\tau \sqrt{\dot{z}^\mu \dot{z}_\mu} \delta^4(x - z(\tau)), \quad (2)$$

We choose the parametrization τ to satisfy $\dot{z}^2 = 1$.

By solving the Heisenberg equation for ϕ , we get the stochastic ALD equation for the charged particle:

$$m\dot{v}^\mu - F^\mu - \frac{e^2}{12\pi} (v^\mu \dot{v}^2 + \ddot{v}^\mu) = -e \vec{\omega}^\mu \phi_h(z) \quad (3)$$

where $v^\mu = \dot{z}^\mu$. The dissipative term corresponds to loss of energy through the radiation and it is called the radiation damping term. On the other hand, the noise term comes from the Unruh effect, namely, interaction of a uniformly accelerated particle with the thermal bath of the radiation field.

We can easily solve the dynamics of small fluctuations of the transverse velocities $v^i = v_0^i + \delta v^i$ in terms of the quantum fluctuations of the field ϕ_h (or its Fourier transformed field φ) as

$$\delta \ddot{v}^i(\omega) = e h(\omega) \partial_i \varphi(\omega), \quad (4)$$

where

$$\delta v^i(\tau) = \int \frac{d\omega}{2\pi} \delta \ddot{v}^i(\omega) e^{-i\omega\tau}, \quad (5)$$

$$\partial_i \phi_h(\tau) = \int \frac{d\omega}{2\pi} \partial_i \varphi(\omega) e^{-i\omega\tau} \quad (6)$$

$$h(\omega) = \frac{1}{-im\omega + \frac{e^2}{12\pi}(\omega^2 + a^2)}. \quad (7)$$

In the following, as an ideal case we consider a uniformly accelerated charged particle in the scalar QED, and investigate the radiation from such a particle. The main issue is the effect of interference.

RADIATION AND INTERFERENCE

Now we calculate the radiation emanated from the uniformly accelerated charged particle. First let's consider the 2-point function of the radiation field. Since the field is written as a sum of the quantum fluctuation (a homogeneous solution) ϕ_h and the inhomogeneous solution in the presence of the charged particle ϕ_I , the 2-point function is given by

$$\begin{aligned} \langle \phi(x) \phi(y) \rangle &= \langle \phi_h(x) \phi_h(y) \rangle \\ &= \langle \phi_I(x) \phi_h(y) \rangle + \langle \phi_h(x) \phi_I(y) \rangle + \langle \phi_I(x) \phi_I(y) \rangle. \end{aligned} \quad (8)$$

The Unruh radiation estimated in [3] is contained in $\langle \phi_I \phi_I \rangle$, which include the Larmor radiation. We need special care

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of the interference terms. As discussed in [5], the interference terms $\langle \phi_I \phi_h \rangle + \langle \phi_h \phi_I \rangle$ may possibly cancel the Unruh radiation in $\langle \phi_I \phi_I \rangle$ after the thermalization occurs. The cancellation is explicitly shown for an internal detector, but it is not obvious whether the same cancellation occurs for the case of a charged particle we are considering.

The inhomogeneous solution of the scalar field is written as

$$\phi_I(x) = e \int d\tau G_R(x - z(\tau)) = \frac{e}{4\pi\rho(x)}. \quad (9)$$

$$\rho(x) = \dot{z}(\tau_-^x) \cdot (x - z(\tau_-^x)), \quad (10)$$

where τ_-^x satisfies $(x - z(\tau_-^x))^2 = 0$, $x^0 > z^0(\tau_-^x)$, which is the proper time of the particle whose radiation travels to the space-time point x . Hence, $z(\tau_-^x)$ lies on an intersection between the particle's world line and the light cone extending from the observer's position x (See Fig 1). We write the superscript x to make the x dependence of τ explicitly.

The particle's trajectory is fluctuating and expressed as $z = z_0 + \delta z + \delta^2 z + \dots$ where we have expanded the trajectory with respect to the interaction with the radiation field (i.e. e). Then ρ is also expanded as $\rho = \rho_0 + \delta\rho + \delta^2\rho + \dots$ and (9) becomes

$$\phi_I = \frac{e}{4\pi\rho_0} \left(1 - \frac{\delta\rho}{\rho_0} + \left(\frac{\delta\rho}{\rho_0} \right)^2 - \frac{\delta^2\rho}{\rho_0} + \dots \right). \quad (11)$$

The first term is the classical potential, but since the particle's trajectory deviates from the classical one, the potential also receives corrections.

Inhomogeneous part

By inserting the expansion (11), the correlator of the inhomogeneous solution ϕ_I becomes

$$\begin{aligned} & \langle \phi_I(x) \phi_I(y) \rangle \\ &= \left(\frac{e}{4\pi} \right)^2 \frac{1}{\rho_0(x)\rho_0(y)} \\ & \times \left(1 + \frac{\langle \delta\rho(x)\delta\rho(y) \rangle}{\rho_0(x)\rho_0(y)} + \frac{\langle (\delta\rho(x))^2 \rangle}{\rho_0^2(x)} + \frac{\langle (\delta\rho(y))^2 \rangle}{\rho_0^2(y)} \right). \end{aligned} \quad (12)$$

The first term gives the Larmor radiation. The other terms correspond to the radiation induced by the fluctuations of the particle's motion.

The calculations of these terms are easy, because one can write $\langle \delta\rho\delta\rho \rangle$ in terms of $\langle \delta\dot{z}^i\delta\dot{z}^i \rangle = \langle \delta v^i\delta v^i \rangle$, which can be obtained by solving the dynamics of fluctuations in the stochastic ALD equation [1, 2]. They become

$$\begin{aligned} & \langle \phi_I(x) \phi_I(y) \rangle \\ &= \left(\frac{e}{4\pi} \right)^2 \frac{1}{\rho_0(x)\rho_0(y)} \left[1 + e^2 \int \frac{d\omega}{2\pi} |h(\omega)|^2 I_S(\omega) \right. \\ & \times \left. \left(\frac{x^i y^i e^{-i\omega(\tau_-^x - \tau_-^y)}}{\rho_0(x)\rho_0(y)} + \frac{x^i x^i}{\rho_0^2(x)} + \frac{y^i y^i}{\rho_0^2(y)} \right) \right]. \end{aligned} \quad (13)$$

Since we are considering the fluctuating motion whose frequency is smaller than the acceleration, I_S can be approximately given by $a^3/12\pi^2$.

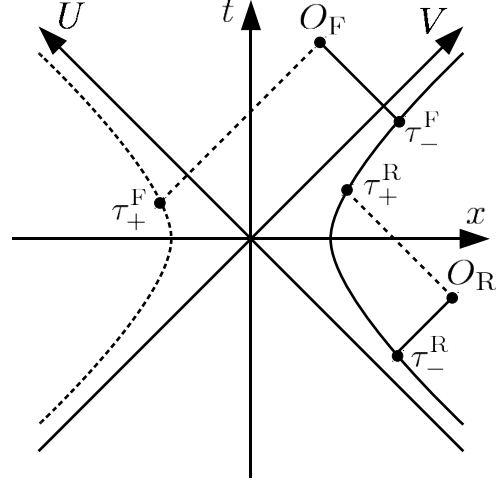


Figure 1: The hyperbolic line in the right wedge denotes the world line of the particle. The points O_F and O_R are observers in the future and right wedges, respectively. For an observer in the right wedge, the light-cone of the observer has two intersections with the world line, and the proper time of the intersections are given by τ_{\pm}^R . For an observer in the future wedge, there is only one intersection on the particle's real trajectory which corresponds to τ_-^F . The other solution $T_+^F = \tau_+^F + i\pi/a$ is complex. One may interpret this complex proper time as the intersection between the light-cone of the observer and the world line of a virtual particle with a real proper time τ_+^F in the left wedge. The superscript letters R or F are used to distinguish two different observers, but we do not use them in the body of the paper to leave the space for the observer's position x .

Interference Term

The calculation of the interference terms is a bit more involved. The inhomogeneous solution ϕ_I is expanded as (11). Since the leading term which has a nonvanishing correlation with the quantum fluctuation ϕ_h is the second term, we have

$$\begin{aligned} & \langle \phi_I(x) \phi_h(y) \rangle + \langle \phi_h(x) \phi_I(y) \rangle \\ &= -\frac{e}{4\pi} \left(\frac{\langle \delta\rho(x) \phi_h(y) \rangle}{\rho_0^2(x)} + \frac{\langle \phi_h(x) \delta\rho(y) \rangle}{\rho_0^2(y)} \right). \end{aligned} \quad (14)$$

The fluctuation of the distance $\delta\rho$ is written in terms of δv^i which is the solution of the stochastic ALD equation (4), and we obtain

$$\langle \phi_h(x) \delta\rho(y) \rangle = -ey^i \int \frac{d\omega}{2\pi} e^{-i\omega\tau_-^y} h(\omega) \langle \phi_h(x) \partial_i \varphi(\omega) \rangle.$$

The integrand can be written as

$$\begin{aligned}\langle \phi_h(x) \partial_i \varphi(\omega) \rangle &= \int d\tau e^{i\omega\tau} \left(\frac{\partial}{\partial y^i} \langle \phi_h(x) \phi_h(y) \rangle \right)_{y=z(\tau)} \\ &= - \int d\tau e^{i\omega\tau} \left(\frac{\partial P(x, \omega)}{\partial x^i} \right),\end{aligned}\quad (15)$$

where

$$P(x, \omega) = \int d\tau \frac{e^{i\omega\tau}}{(x^0 - z^0(\tau) - i\epsilon)^2 - (x^1 - z^1(\tau))^2 - x_\perp^2}.$$

$x_\perp^2 = (x^2)^2 + (x^3)^2$ is the transverse distance. The τ integral can be calculated by the contour integral. The residues are located where the invariant length between the observed point x and a point on the particle's trajectory vanishes. The condition is nothing but the condition that the radiation field propagates on the light cone. Fig.1 shows such a situation. It is interesting that the condition for residues has a solution on an intersection of the light-cone of the observer and the virtual path of a particle (dotted line in the left wedge). We skip the calculations and show the final results of the interference terms;

$$\begin{aligned}\langle \phi_I(x) \phi_h(y) \rangle + \langle \phi_h(x) \phi_I(y) \rangle &= \frac{-iae^2 x^i y^i}{(4\pi)^2 \rho_0(x)^2 \rho_0(y)^2} \int \frac{d\omega}{2\pi} \frac{1}{1 - e^{-2\pi\omega/a}} \\ &\times \left[e^{-i\omega(\tau_-^x - \tau_-^y)} h(-\omega) \left(\frac{aL_x^2}{2\rho_0(x)} - \frac{i\omega}{a} \right) \right. \\ &\quad e^{-i\omega(\tau_-^x - \tau_-^y)} - h(\omega) \left(\frac{aL_y^2}{2\rho_0(y)} + \frac{i\omega}{a} \right) \\ &\quad + e^{-i\omega(\tau_+^x - \tau_+^y)} h(-\omega) \left(-\frac{aL_x^2}{2\rho_0(x)} - \frac{i\omega}{a} \right) Z_x(-\omega) \\ &\quad \left. - e^{-i\omega(\tau_-^x - \tau_+^y)} h(\omega) \left(-\frac{aL_y^2}{2\rho_0(y)} + \frac{i\omega}{a} \right) Z_y(-\omega) \right]\end{aligned}\quad (16)$$

where

$$Z_x(\omega) = e^{\pi\omega/a} \theta(x^0 - x^1) + \theta(x^1 - x^0) \quad (17)$$

$$L_x^2 = -x^\mu x_\mu + \frac{1}{a^2}, \quad L_y^2 = -y^\mu y_\mu + \frac{1}{a^2}. \quad (18)$$

Partial Cancellation

The correlation function of the inhomogeneous terms (13) depends only on τ_- . The interference terms contain both of terms depending on τ_- and τ_+ ; the first term in the parenthesis of (17) depends only on τ_- , so it is the term that may cancel the inhomogeneous terms (i.e. the Unruh radiation). Using the relation

$$h(\omega) + h(-\omega) = \frac{e^2}{6\pi} (\omega^2 + a^2) |h(\omega)|^2, \quad (19)$$

one can show that a part of the interference terms

$$\begin{aligned}&\frac{iae^2 x^i y^i}{(4\pi)^2 \rho_0(x)^2 \rho_0(y)^2} \\ &\times \int \frac{d\omega}{2\pi} \frac{e^{-i\omega(\tau_-^x - \tau_-^y)}}{1 - e^{-2\pi\omega/a}} \left(h(-\omega) \frac{i\omega}{a} + h(\omega) \frac{i\omega}{a} \right)\end{aligned}\quad (20)$$

cancels the first correction term of the inhomogeneous part in (13). This term was obtained by taking a derivative of $e^{i\omega\tau_-}$ in $P(x, \omega)$. But note that the cancellation occurs only partially. Furthermore, the τ_+ -dependent terms in the interference terms cannot be canceled with the Unruh radiation.

The Energy Momentum Tensor

Given the 2-point function, we can calculate the energy momentum tensor of the radiation

$$\langle T_{\mu\nu}(x) \rangle = \langle : \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g_{\mu\nu} \partial^\alpha \phi \partial_\alpha \phi : \rangle. \quad (21)$$

It is a sum of the classical and the fluctuation parts; $T_{\mu\nu} = T_{cl, \mu\nu} + T_{fluc, \mu\nu}$. The classical part is given by

$$T_{cl, \mu\nu} \sim \frac{e^2 \partial_\mu \rho_0 \partial_\nu \rho_0}{(4\pi)^2 \rho_0^4}. \quad (22)$$

It corresponds to the energy momentum tensor of the Larmor radiation. From (10) it can be seen to be proportional to a^2/r^2 where a is the acceleration and r is the spacial distance from the particle to the observer. $T_{fluc, \mu\nu}$ is the energy momentum of the additional radiation

$$\begin{aligned}T_{fluc, \mu\nu} &= \frac{(x^i)^2}{\rho_0^2} \left[\left(\frac{e^2}{\pi} I_m - \frac{6ma^2 I_1 L_x^2}{\rho_0} \right) T_{cl, \mu\nu} \right. \\ &\quad - \frac{e^2 a^2 L_x^2}{(4\pi)^2 \rho_0^3} \left(m I_3 \partial_\mu \tau_-^x \partial_\nu \tau_-^x + \frac{2m I_1}{\rho_0 L_x^2} (x_\mu \partial_\nu \rho_0 + x_\nu \partial_\mu \rho_0) \right) \\ &\quad + \frac{e^2 I_m}{12\pi L_x^2} (x_\mu \partial_\nu \tau_-^x + x_\nu \partial_\mu \tau_-^x) \\ &\quad \left. - \frac{e^2 I_m}{24\pi \rho_0} (\partial_\mu \tau_-^x \partial_\nu \rho_0 + \partial_\nu \tau_-^x \partial_\mu \rho_0) \right]\end{aligned}\quad (23)$$

where $I_1 = \frac{3}{2mae^2}$, $I_3 \sim \Omega_-^2 I_1 \ll a^2 I_1$, $I_m = I_3 + a^2 I_1 \sim a^2 I_1$. Hence, these terms originating from the fluctuating motion of the particle is proportional to a^3 , and smaller by a factor of a compared to the above Larmor radiation. Though they have different angular distribution, there is an overall factor (x_i^2) in front and they vanish at the forward direction. Together with the long relaxation time discussed in [2], the detection of the Unruh radiation seems to be very difficult experimentally.

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